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Chiral restoration in effective quark models with non-local interactions ⋆

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Chiral restoration at finite temperatures is studied in chiral quark models with non-local regulators. At the leading-N_c level we find transition temperature of the order 100MeV. Meson-loop contributions are also analyzed and found to have a very small effect.

Recently, considerable activity has been focused on chiral quark models with *non-local* interactions [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17]. In this approach interactions depend on the momenta carried by the quarks, which leads to a momentum-dependent quark mass, generated by spontaneous breaking of the chiral symmetry. There are several important reasons why it is worthwhile to consider such models: (1) Non-locality arises naturally in the most sound and successful approaches to low-energy quark dynamics, namely the instanton-liquid model [18,19,20] and the Schwinger-Dyson resummation techniques [21]. Hence, we should deal with non-local regulators from the outset and models with local interactions (the Nambu–Jona-Lasinio model and its progeny) may be viewed as simplifications to the non-local models. (2) The non-local regularization leads to preservation of anomalies [6,22] and to correct charge quantization [15,16]. With local methods, such as the proper-time regularization or the quark-loop momentum cut-off [9,10,23,24] the matching of anomalies can only be achieved if the finite (anomalous) part of the Euclidean action is left unregularized, and the infinite (non-anomalous) part is regularized. When both parts are regularized, anomalies are violated substantially [25,26] and the theory is not consistent. The different treatment of the anomalous and non-anomalous parts of the action is rather artificial and it is quite appealing that with non-local regulators both parts of the action can be treated in a

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unified way. (3) With non-local interactions the effective action is finite to all orders in the loop expansion (*i.e.* the $1/N_c$ expansion). In particular, the meson loops are finite and it is not necessary to introduce additional cut-offs, as was the case of local models [27,28,29]. As a result, the non-local models have more predictive power. (4) In the baryonic sector the non-local models are capable of producing stable solitons [15] without the extra constraint that forces the σ and π fields to lie on the chiral circle. Such a constraint is external to the known derivations of effective chiral quark models.

In view of the above-listed advantages of the non-local models over the conventional Nambu–Jona-Lasinio-like models it is desired to check other applications of non-local effective chiral theories. In this paper we investigate the restoration of chiral symmetry at finite temperatures and vanishing baryon density. The basic quantity of our study is the quark condensate, which in the chiral limit of the vanishing current quark mass is the order parameter of chiral symmetry. We show that chiral restoration is a second-order phase transition, as in the local models, and it occurs at temperatures $T_c \sim 100$MeV, which is somewhat less than in local theories, where the typical T_c is around 150MeV. We also investigate the dependence of the critical temperature T_c on the detailed shape of the non-local regulator and find it to be weak. With a finite current quark mass the chiral transition is, as usual, a smooth cross-over. In addition, we show that the effects of meson-loops (higher-order effects in the $1/N_c$ expansion) of the quark condensate are very small in non-local models.

We begin with a brief review of the model. Our starting point is the two-flavor effective non-local Euclidean action in the bosonized form

$$I = -\mathrm{Tr}\ln\left(-i\gamma_\mu\partial^\mu + m + r\Phi\Gamma_\Phi r\right) + \tfrac{a^2}{2}\int d^4x(\Phi_0^2 + \mathbf{\Phi^2}), \tag{1}$$

where $\frac{1}{a^2}$ has the meaning of the coupling constant of the four-quark interactions, m is the current quark mass, $\Phi = (\Phi_0, \Phi_a)$ are the mean meson fields with quantum numbers of the σ meson and the pions, respectively. The fields Φ are local in coordinate space. Matrices $\Gamma_\Phi = (\Gamma_{\Phi_0}, \Gamma_{\Phi_a})$ are acting in spin and flavor spaces, with $\Gamma_{\Phi_0} = 1$ and $\Gamma_{\Phi_a} = i\gamma_5\tau^a$. The regulator operator r is local in momentum space, *i.e.* $\langle p|r|p'\rangle = \delta(p-p')r(p)$. It introduces a cut-off scale Λ, limiting appropriately momenta of the quarks. The symbol Tr denotes the full trace, *i.e.* the functional trace and the matrix traces over the Dirac, flavor and color degrees of freedom.

Through the use of the Euler-Lagrange equations for the Φ field it is straightforward to show that the action (1) is equivalent to the model where the four-quark interaction has a separable form [7,8], namely

$$L_{\rm int} = -\frac{1}{2a^2}\int\frac{d^4p_1}{(2\pi)^4}\int\frac{d^4p_2}{(2\pi)^4}\int\frac{d^4p_3}{(2\pi)^4}\int\frac{d^4p_4}{(2\pi)^4}\times \tag{2}$$

$$\delta^4(p_1+p_2+p_3+p_4)\overline{\psi}(p_1)r(p_1)\Gamma_\Phi r(p_2)\psi(p_2)\overline{\psi}(p_3)r(p_3)\Gamma_\Phi r(p_4)\psi(p_4).$$

This form has a very clear interpretation via Feynman diagrams: the quark vertices carry a regulator on each quark line entering the vertex.

In the instanton-liquid model the regulator can be evaluated for low Euclidean momenta. It has the form [18,19,20]

$$r\left(p^2\right) = -\frac{z}{2}\frac{d}{dz}\left(I_0\left(z\right)K_0\left(z\right) - I_1\left(z\right)K_1\left(z\right)\right), \quad z = \frac{\sqrt{p^2}\rho}{2}, \tag{3}$$

where I and K are the modified Bessel functions, and ρ is the average instanton size, of the order of $(600\mathrm{MeV})^{-1}$. We introduce $\Lambda = 2/\rho$, which is therefore of the order of 1.2GeV. We will also study an approximate formula for the regulator [7,8], given by the Gaussian function:

$$r\left(p^2\right) = \exp\left(-\frac{p^2}{2\Lambda^2}\right) \tag{4}$$

We first analyze the model at the one-quark-loop level. In the vacuum the values of the fields Φ_0 and Φ_a are obtained from the stationary-point conditions: $\frac{\delta I}{\delta \Phi^a} = 0$ and $\frac{\delta I}{\delta \Phi_0} = 0$. The first condition is trivially fulfilled for $\Phi_a = 0$, and the second one has a nontrivial solution $\Phi_0 = S_0$, which satisfies the equation

$$a^2 = 2\nu\left(g\left(S_0\right) + \frac{m}{S_0}g'\left(S_0\right)\right), \tag{5}$$

with $g\left(S_0\right)$ and $g'\left(S_0\right)$ given by the following integrals:

$$g\left(S_0\right) = \int \frac{d^4k}{(2\pi)^4}\frac{R^2(k^2)}{D(k^2)} \tag{6}$$

$$g'\left(S_0\right) = \int \frac{d^4k}{(2\pi)^4}\frac{R(k^2)}{D(k^2)} \tag{7}$$

Above, we have used the notation

$$\begin{aligned} &\nu = 2N_cN_f, \quad D(k^2) = k^2 + \mathcal{M}^2(k^2), \\ &\mathcal{M}(k^2) = R(k^2)S_0 + m, \quad R(k^2) = r^2(k^2), \end{aligned} \tag{8}$$

where $N_c = 3$ is the number of colors and $N_f = 2$ is the number of flavors. The quark condensate (for a single flavor) is defined as $\langle \overline{q}q\rangle \equiv \frac{1}{N_f}\frac{\delta I}{\delta m}$, which explicitly gives

$$\langle \overline{q}q\rangle = -4N_c\int \frac{d^4k}{(2\pi)^4}\frac{\mathcal{M}(k^2)}{D(k^2)}. \tag{9}$$

If $m \neq 0$, the above expression is ultraviolet divergent. However, in this case one should subtract the perturbative vacuum part from (9), *i.e.* the piece with $S_0 = 0$. The results is well-defined:

$$\begin{aligned}
\langle \overline{q}q \rangle \longrightarrow \langle \overline{q}q \rangle - \langle \overline{q}q \rangle_{S_0=0} &= -4N_c \int \frac{d^4k}{(2\pi)^4} \left(\frac{\mathcal{M}(k^2)}{k^2 + \mathcal{M}^2(k^2)} - \frac{m}{k^2+m^2} \right) = \\
&= -4N_c S_0 g'(S_0) + 4N_c m \int \frac{d^4k}{(2\pi)^4} \frac{\left(2m + R(k^2)S_0\right) R(k^2) S_0}{D(k^2)\left(k^2+m^2\right)} = \\
&= -4N_c S_0 g'(S_0) + \mathcal{O}(m) \qquad (10)
\end{aligned}$$

The model has three parameters: the cut-off Λ, the current quark mass m and the coupling constant $\frac{1}{a^2}$. Two conditions for parameters are obtained by demanding that the pion decay constant, F_π, and the pion mass, m_π, assume their physical values: $F_\pi = 93$MeV, and $m_\pi = 139$MeV. To leading order in m the expressions for these quantities in our model are [7,8]:

$$F_\pi{}^2 = \nu S_0^2 \int \frac{d^4k}{(2\pi)^4} \frac{R^2(k^2) - k^2 R(k^2) \frac{dR(k^2)}{d(k^2)} + k^4 \left(\frac{dR(k^2)}{d(k^2)}\right)^2}{D^2(k^2)}\,, \qquad (11)$$

$$m_\pi^2 = \frac{2\nu S_0 m g'(S_0)}{F_\pi^2}. \qquad (12)$$

We note that the elimination of $g'(S_0)$ from r.h.s of the expression (12), with help of the equation (10), leads to

$$m_\pi^2 = -\frac{m \langle \overline{q}q \rangle}{F_\pi^2}, \qquad (13)$$

in which we recognize as the Gell-Mann-Oakes-Renner relation. One parameter of the model is left free. Through the use of the stationary-point condition we can trade it for the value of S_0, which now is treated as the only remaining parameter. In principle we could still fit it, making use another helpful quantity, namely the gluon condensate. In our model it can be written as [30]

$$\left\langle \frac{\alpha_s G^{\mu\nu} G_{\mu\nu}}{\pi} \right\rangle = \frac{8}{N_f} a^2 S_0^2 - m \langle \overline{q}q \rangle. \qquad (14)$$

However, the dependence of the gluon condensate on S_0 (with F_π and m_π fixed at physical values) is very weak, which makes the fit impossible in practice due to a large uncertainty in our knowledge of $\langle \alpha_s/\pi\, G^{\mu\nu} G_{\mu\nu} \rangle$. Reasonable parameters yield the value of the quark condensate $\langle \overline{q}q \rangle \sim (250\text{MeV})^3$, the current quark mass $m \sim 5$MeV and the gluon condensate $\langle \alpha_s/\pi\, G^{\mu\nu} G_{\mu\nu} \rangle \sim (330 \pm 30\text{MeV})^4$. This favors S_0/Λ in the range $0.2 - 0.5$.

Now we pass to the analysis of the temperature dependence of the quark condensate. For this purpose we use the Matsubara formalism, which leads to

the following simple thermalization rule for one-quark-loop integrals

$$\int \frac{d^4k}{(2\pi)^4} f(k) = \int \frac{dk_0}{2\pi} \int \frac{d^3k}{(2\pi)^3} f(k_0, \vec{k}) \longrightarrow T \sum_{j=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} f(E_j, \vec{k}), \quad (15)$$

where T is the temperature, f is the integrand of the quark loop, and k denotes the momentum running around the loop. The summation runs over the appropriate fermionic Matsubara frequencies, $E_j = (2j+1)\pi T$. After thermalization and integration of the angular dependence, the equations (6) and (7) acquire the following finite-temperature form:

$$g_T(S_0) = \frac{1}{\pi^2} T \sum_{n=0}^{\infty} \int dk \frac{k^2 R^2(k^2+E_n^2)}{k^2+E_n^2+\mathcal{M}^2(k^2+E_n^2)}, \quad (16)$$

$$g'_T(S_0) = \frac{1}{\pi^2} T \sum_{n=0}^{\infty} \int dk \frac{k^2 R(k^2+E_n^2)}{k^2+E_n^2+\mathcal{M}^2(k^2+E_n^2)}. \quad (17)$$

Of course the value of S_0 depends on T via the finite-temperature stationary-point condition:

$$a^2 = 2\nu \left(g_T(S_0) + \frac{m}{S_0} g'_T(S_0) \right), \quad (18)$$

where now k denotes the length of the 3-momentum $\vec{k}$. The finite-temperature expression for the quark condensate is

$$\begin{aligned} \langle \overline{q} q \rangle_T = & -4N_c S_0 g'_T(S_0) + \quad (19) \\ & \frac{4N_c m}{\pi^2} T \sum_{n=0}^{\infty} \int dk \frac{k^2 \left(2m + R(k^2+E_n^2) S_0\right) R(k^2+E_n^2) S_0}{\left(k^2+E_n^2+\mathcal{M}(k^2+E_n^2)^2\right)\left(k^2+E_n^2+m^2\right)}, \end{aligned}$$

There is an important remark to be made in this place. The described way of thermalization, typically done in effective chiral quark models, tacitly assumes that the model parameters (here a and Λ) do not explicitly depend on T. There is no sound reason for this assumption in our effective theory, except for simplicity. Allowing temperature dependence of model parameters influences the values of T_c. We will return to this question shortly.

Table 1 shows our numerical results, with the meaning of various quantities given in the caption. The striking feature here is the very weak dependence of the critical temperature T_c on the model parameters and on the regulator used. We find $T_c \sim 100$MeV, which is somewhat less than in conventional Nambu–Jona-Lasinio model, where it is of the order of $120-150$MeV. The reason for this lower value is the following. The integrand of the expression for g_T in local models is simply $k^2/(k^2+E_n^2+M^2)$, where M is the quark mass, instead of our expression $k^2 R^2(k^2+E_n^2)/(k^2+E_n^2+M^2R^2(k^2+E_n^2))$. The presence of the regulator function R causes the much more rapid decrease

Table 1
Results for the two considered regulators and various parameter sets. The first column is the ratio S_0/Λ, the next three columns display the values of the model parameters Λ, a, and m, adjusted in such a way that $F_\pi = 93$MeV and $m_\pi = 139$MeV for the given S_0/Λ. The next two columns show the calculated values of S_0 and $\langle -\overline{q}q\rangle^{\frac{1}{3}}$ at $T = 0$. The last three columns give the results obtained in the strict chiral limit, with m set to 0, and Λ and a from columns (2–3). The quantities $S_{0,m=0}$ and $\langle -\overline{q}q\rangle^{\frac{1}{3}}_{m=0}$ denote the values of the scalar field and the quark condensate for this case and for $T = 0$, and $T_{\rm crit}$ denotes the critical temperature of the second-order chiral phase transition.

$\frac{S_0}{\Lambda}$	Λ MeV	a MeV	m MeV	S_0 MeV	$\langle -\overline{q}q\rangle^{\frac{1}{3}}$ MeV	$S_{0,m=0}$ MeV	$\langle -\overline{q}q\rangle^{\frac{1}{3}}_{m=0}$ MeV	$T_{\rm crit}$ MeV
Instanton-model regulator								
0.2	1506	219	3.5	301	287	256	274	102
0.3	1191	165	4.9	357	256	310	246	101
0.4	1019	134	6.2	408	238	358	229	100
0.5	907	115	7.3	454	225	401	218	99
Gaussian regulator								
0.2	1155	296	3.9	231	290	192	278	106
0.3	894	218	5.9	268	250	230	243	105
0.4	754	176	7.7	302	227	263	221	104
0.5	666	149	9.4	333	211	295	207	103

of the integrand with the temperature T, which enters through the Matsubara energies E_n. The more rapid decrease of g_T with T leads directly to a lower critical temperature T_c as compared to the local models. One should bare in mind here that any explicit dependence of a on T, not considered here, would influence the values of T_c. For instance, an increase of a by 20% (equivalent to decreasing the coupling constant of the four-quark interaction) leads to an increase of T_c to values around 120MeV. We note that calculations in instanton-based models of the QCD vacuum [19,31,32,33] yield values of T_c in the range 125–175MeV.

To summarize this part, we note that at the one-quark-loop level the behavior of the non-local model is qualitatively not different from the local models. We find the chiral phase transition, which is second-order in the strict chiral limit, and the corresponding critical temperature is very weakly dependent on the details of the model.

In the remaining part of this paper we consider meson loops effects. Such effects, although suppressed by $1/N_c$, are expected to be important due to

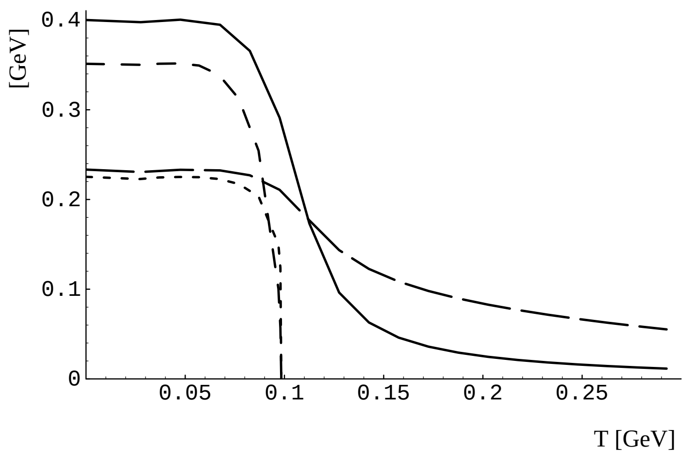

Fig. 1. The temperature dependence of S_0 and $\langle -\overline{q}q\rangle^{\frac{1}{3}}$ for the instanton-liquid-model regulator with $a = 134\text{MeV}$, $\Lambda = 1019\text{MeV}$ and $m = 6.19\text{MeV}$ or $m = 0$. The meaning if the curves is as follows: S_0 – solid line, $\langle -\overline{q}q\rangle^{\frac{1}{3}}$ – long-dash line, $S_{0,m=0}$ – medium-dash line, $\langle -\overline{q}q\rangle^{\frac{1}{3}}_{m=0}$ – short-dash line. Note considerable difference between the strict chiral limit and the finite-m case.

the low mass of the pion. We follow the method described in Refs. [27,28], where the meson loops have been introduced in effective chiral models in a consistent (*i.e.* symmetry-conserving) way. As mentioned in the introduction, conventional Nambu–Jona-Lasinio models require an extra regulator for the meson loop – making the quark loop finite via momentum cut-off, the proper-time or Pauli-Villars method, still leaves the momentum in the meson loop unconstrained. With non-local regulators this is no longer case. The interactions (2) cut-off any momentum flowing through a quark vertex. As a result, calculation of a diagram with any number of loops is finite.

In particular, meson loops are finite, and we now incorporate them in our study. First we analyze the problem at $T = 0$. The meson-loop contribution in the effective action can be obtained in a standard way by integrating out meson fluctuations around the stationary-point configuration [28]. The new stationary-point condition for S_0 assumes the form

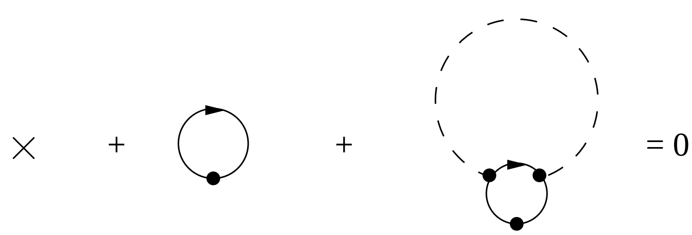

Fig. 2. The diagrams contributing to the stationary-point equation (20) with meson loops included. The dashed line corresponds to the meson propagators K_π and K_σ defined in Eq. (23). The cross denotes the term $a^2 S_0$.

$$
S_0 \left(a^2 - 2\nu g(S_0) - 2\nu \frac{m}{S_0} g'(S_0) \right)
+ \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \left(V_\sigma(p) K_\sigma(p) + N_B V_\pi(p) K_\pi(p) \right) = 0 \ , \tag{20}
$$

where $N_B = N_f^2 - 1 = 3$ is the number of pions, K_σ and K_π are meson propagators at the quark-loop level and the 3-point quark loops (see Fig. 2) are equal to:

$$
\begin{aligned}
V_\sigma(p) = & -2\nu \int \frac{d^4k}{(2\pi)^4} \frac{R^2(p) R(k+p)}{D^2(k) D(k+p)} (-2\mathcal{M}(k)\, k \cdot (k+p) \\
& - \mathcal{M}(k+p)(k^2 + \mathcal{M}^2(k))) \qquad (21) \\
V_\pi(p) = & -2\nu \int \frac{d^4k}{(2\pi)^4} \frac{R^2(k) R(k+p)}{D^2(k) D(k+p)} (-2\mathcal{M}(k)\, k \cdot (k+p) \\
& + \mathcal{M}(k+p)(k^2 - \mathcal{M}^2(k))) \qquad (22)
\end{aligned}
$$

The inverse meson propagators can be written as:

$$
\begin{aligned}
K_\pi^{-1}(p) &= \nu p^2 f_\pi(p) + \frac{m}{S_0} 2\nu g'(S_0) \ , \\
K_\sigma^{-1}(p) &= K_\pi^{-1}(p) + 4\nu S_0^2 f_\sigma(p) \ . \qquad (23)
\end{aligned}
$$

where we have introduced

$$
f_\pi(p) = -\frac{2}{p^2} \int \frac{d^4k}{(2\pi)^4} \left(\frac{R(k) R(k+p)(k \cdot (k+p) + \mathcal{M}(k)\mathcal{M}(k+p))}{D(k) D(k+p)} - \frac{R^2(k)}{D(k)} \right) \tag{24}
$$

and

$$
f_\sigma(p) = \frac{1}{S_0^2} \int \frac{d^4k}{(2\pi)^4} \frac{R(k) R(k+p) \mathcal{M}(k) \mathcal{M}(k+p)}{D(k) D(k+p)} \ . \tag{25}
$$

All terms included in the stationary-point condition are shown in Fig. 2. A similar expression can be obtained for the quark condensate.

The results are somewhat surprising. We have found large cancellations in the meson-loop contribution, such that the net correction to the value of S_0 is less than 1%, and to $\langle \bar{q} q \rangle$ of the order of 3-4% on top of the quark-loop value. To be more precise, for the Gaussian regulator with the parameters $\Lambda = 754\text{MeV}$, $a = 176\text{MeV}$, and $m = 0$ we find $\langle \bar{q} q \rangle = (0.0505 + 0.0021 - 0.0041)\text{GeV}^3$, where we have decomposed in the quark-loop, σ-loop, and the pion-loop contribution, respectively. We note the cancellation between the σ- and pion-loop contributions. The phenomenon occurs for various regulators and parameter sets. It is also instructive to look at the integrand of the p-integration in Eq.

(20). After carrying the angular integration, we are left with a radial integration over $|\vec{p}|$. The integrand in the pion-loop case changes sign as we increase $|\vec{p}|$, and this results in an additional cancellation. At large values of $|\vec{p}|$ the integrand drops to zero. This behavior is quite different from the calculations in the local model, as in Ref. [28]. There the analogous integrands in the meson-loop contributions increase with $|\vec{p}|$, and the results depend strongly on the value of the momentum cut-off in the meson loop.

Having found small meson-loop effects for $T = 0$ we expect them to remain small at finite T. This is indeed the case. The expression (20) and the similar expression for the quark condensate can be thermalized by the prescription (15). The only difference now is that in the mesonic loop we have to use bosonic Matsubara frequencies, *i.e.* in the meson-loop we replace

$$\int \frac{d^4p}{(2\pi)^4} f(p) = \int \frac{dp_0}{2\pi} \int \frac{d^3p}{(2\pi)^3} f(p_0, \vec{p}) \longrightarrow T \sum_{j=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3} f(\omega_j, \vec{p}), \quad (26)$$

with $\omega_j = 2\pi j T$. The numerical calculations show that the effect of the meson loops for finite temperatures is tiny, and the quark loop is dominant. The critical temperatures are practically unchanged. This is different than in local models, where the meson loops have a substantial effect on the finite-T behavior of the model [29].

Our main conclusions are as follows: the restoration of chiral symmetry in non-local chiral quark models proceeds similarly to the conventional Nambu–Jona-Lasinio models. In the strict chiral limit the transition is second order, however the critical temperatures are lower, around 100MeV. For finite current quark masses we find a smooth cross-over to the restored phase. An interesting result is a very small contribution of meson loops to the quark condensate, which is in contrast to the calculations within the Nambu–Jona-Lasinio model. The effect occurs for a variety of regulators used. It is encouraging, since it indicates that we can rely on the simple-to-do one-quark-loop calculations. It remains to be seen whether in non-local models the meson loop effects are also negligible for meson correlators and other Green's functions.

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